2025 Simons Lectures in Mathematics

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Tales from the Crossroads Where Dynamics, Arithmetic, and Geometry Meet

Discrete dynamics is the study of orbits of points under iteration of a self-map $f: X \to X$, where classically the space X would be $\mathbf{R}$ or $\mathbf{C}$ or a real or complex manifold. Arithmetic dynamics studies orbits of points and maps that are of arithmetic interest: for example X could be $\mathbf{Q}$ or a finite extension K of $\mathbf{Q}$, and f might be a polynomial or rational function. Thus arithmetic dynamics combines dynamics (iteration) of algebro-geometric functions (polynomial and rational maps) applied to number theoretically interesting points (having rational or algebraic coordinates).

The first lecture – largely involving only undergraduate mathematics - will describe a dictionary between discrete dynamics and arithmetic geometry and will show how it leads to conjectures and results that motivate the relatively new field of arithmetic dynamics. The second will center around a recent conjecture concerning the spacing of orbits of arithmetically interesting orbits, which itself was motivated by an old conjecture on the growth-rate of rational points on algebraic varieties: we will describe the two conjectures and sketch proofs (joint work with Hector Pasten) of the dynamical conjecture in several cases. The theme of the third lecture is families of dynamical systems, especially the family of degree d rational self-maps of $\mathbf{P}^1$. We will describe the associated dynamical moduli space Mod_d of such maps and discuss recent results of Ingram and others that give a fundamental relationship between the arithmetic complexity of a map and the arithmetic complexity of its critical points.

Lecture 1. Arithmetic Dynamics: A Survey of Motivating Problems and Results
Tuesday, March 25, 2025 – 2:15 pm, SCGP 103

Lecture 2. Are Orbits of Rational Points Widely Spaced?
Wednesday, March 26, 2025 – 4:00 pm, SCGP 103

Lecture 3. Dynamical Moduli Spaces, Critical Points, and Canonical Heights
Thursday, March 27, 2025 – 2:15 pm, SCGP 103